QUANTITATIVE METHODS FOR MANAGERS

UNIT-1 MATRICES & DETERMINANTS

INTRODUCTION TO MATRICES & DETERMINANTS

Matrix definition

A matrix is a set of numbers arranged in rows and columns so as to form a rectangular array. The numbers are called the elements, or entries, of the matrix.

Examples of Matrix

$$\begin{bmatrix} \boldsymbol{a11} & \boldsymbol{a12} \ldots\ldots & \boldsymbol{a1n} \\ \boldsymbol{a21} & \boldsymbol{a22} \ldots\ldots & \boldsymbol{a2n} \\ \ldots. & \ldots & \boldsymbol{amn} \end{bmatrix}$$

Application of Matrix

Matrices have wide applications in engineering, physics, economics, and statistics as well as in various branches of mathematics.

TYPES OF MATRIX:

- Row matrix
- Column matrix
- Null matrix or Zero matrix
- Square Matrix
- Diagonal matrix
- Upper Triangular matrix

- Lower Triangular matrix
- Unit matrix

1) **Row Matrix:**

A row matrix is a type of matrix that has a single row.

Example:

$$A = \begin{pmatrix} a11 & a12 & a13 & \ldots & a1n \end{pmatrix}$$

2) **Column matrix:**

A column matrix is a type of matrix that has only one column.

Example:

$$\begin{pmatrix} \mathbf{a\ 11} \\ \mathbf{a21} \\ \ldots \\ \mathbf{a\ n1} \end{pmatrix}$$

3) **Null matrix or Zero matrix:**

A zero matrix or null matrix is a matrix all of whose entries are zero.

Example:

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

4) **Square Matrix:**

A square matrix is a matrix with the same number of rows and columns.

Example:

$$\begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{3} \end{bmatrix}$$

5) **Diagonal Matrix:**

A diagonal matrix is a matrix in which the entries outside the main in which the entries outside the main diagonal are all zero; the term usually refers to square matrices.

Example:

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{4} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{8} \end{bmatrix}$$

6) **Upper Triangular matrix:**

A square matrix is called upper triangular if all the entries below the main diagonal are zero.

Example:

$$\begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{5} & \mathbf{8} \\ \mathbf{0} & \mathbf{6} & \mathbf{5} & \mathbf{4} \\ \mathbf{0} & \mathbf{0} & \mathbf{6} & \mathbf{2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{8} \end{bmatrix}$$

7) Lower triangular matrix:

A square matrix is called lower triangular if all the entries above the main diagonal are zero.

Example:

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{5} & \mathbf{6} & \mathbf{0} & \mathbf{0} \\ \mathbf{8} & \mathbf{1} & \mathbf{6} & \mathbf{0} \\ \mathbf{2} & \mathbf{7} & \mathbf{4} & \mathbf{8} \end{bmatrix}$$

8) Unit Matrix:

A unit matrix can be defined as a scalar matrix in which all the diagonal elements are equal to 1 and all the other elements are zero.

Example:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

DETERMINANTS:

A determinant is a square array of numbers (written within a pair of vertical lines) which represents a certain sum of products.

USES OF DETERMINANTS:

The determinant is useful for solving linear equations, capturing how linear transformation change area or volume, and changing variables in integrals.

Example:

$$A = \begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 2 & 4 \\ -3 & 1 \end{vmatrix} = (2\times 1) - (4\times(-3)) = 2 + 12 = 14$$

ADDITION OF MATRIX: In mathematics, matrix addition is the operation of adding two matrices by adding the corresponding entries together.For example

$$A = \begin{pmatrix} 5 & -5 & 3 \\ -5 & 4 & 3 \\ -4 & 3 & 1 \\ 3 & -4 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -3 & -4 & 0 \\ 3 & 2 & 3 \\ -2 & -4 & 2 \\ -3 & -1 & -1 \end{pmatrix}$$

$$A + B = \begin{pmatrix} (5 + -3) & (-5 + -4) & (3 + 0) \\ (-5 + 3) & (4 + 2) & (3 + 3) \\ (-4 + -2) & (3 + -4) & (1 + 2) \\ (3 + -3) & (-4 + -1) & (1 + -1) \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -9 & 3 \\ -2 & 6 & 6 \\ -6 & -1 & 3 \\ 0 & -5 & 0 \end{pmatrix}$$

SUBTRACTION OF MATRIX:

Subtraction of matrices is an operation of element-wise subtraction of matrices of the same order, that is, matrices that have the same number of rows and columns. In subtracting two matrices, we subtract the elements in each row and column from the respective elements in the row and column of the other matrix.

The number of rows and columns should be the same for the matrix subtraction. The subtraction of a matrix from itself results in a null matrix, that is, A-A=0. Subtraction of matrices is the addition of the negative of a matrix to another matrix, that is A-B=A+ (-B).

Example:

2x2 order matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ and } B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$A - B = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} \\ a_{21} - b_{21} & a_{22} - b_{22} \end{bmatrix}$$

$$A = \begin{bmatrix} 12 & -3 \\ 2 & 15 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 1 \\ 11 & -8 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 12 - 6 & -3 - 1 \\ 2 - 11 & 15 - (-8) \end{bmatrix} = \begin{bmatrix} 6 & -4 \\ -9 & 23 \end{bmatrix}$$

3x3 order matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \text{ and } B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$A - B = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} & a_{13} - b_{13} \\ a_{21} - b_{21} & a_{22} - b_{22} & a_{23} - b_{23} \\ a_{31} - b_{31} & a_{32} - b_{32} & a_{33} - b_{33} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1-9 & 2-8 & 3-7 \\ 4-6 & 5-5 & 6-4 \\ 7-3 & 8-2 & 9-1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -6 & -4 \\ -2 & 0 & 2 \\ 4 & 6 & 8 \end{bmatrix}$$

MULTIPLICATION OF MATRIX:

The definition of matrix multiplication indicates a row-by-column multiplication, where the entries in the ith row of A are multiplied by the corresponding entries in the jth column of B and then adding the results.

Matrix multiplication is NOT commutative. If neither A nor B is an identity matrix, AB≠BA

Law of Matrix Multiplication:

The product of two matrices will be defined if the number of columns in the first matrix is equal to the number of rows in the second matrix. If the product is defined, the resulting matrix will have the same number of rows as the first matrix and the same number of columns as the second matrix.

Types of Multiplication Matrix:

There are two types of multiplication for matrices: scalar multiplication and multiplication matrix. scalar multiplication is easy. You just take a regular number (called a "scalar") and multiply it on every entry in the matrix.

Scalar matrix:

- **For the following matrix A, find $2A$ and $-1A$.**

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

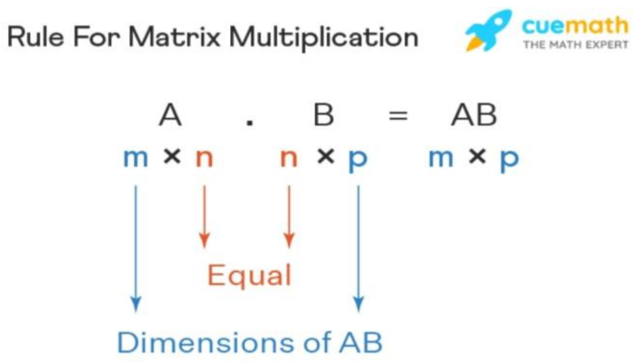
To do the first scalar multiplication to find $2A$, I just multiply a 2 on every entry in the matrix:

$$2A = 2 \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 & 2\cdot 2 \\ 2\cdot 3 & 2\cdot 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

The other scalar multiplication, to find $-1A$, works the same way:

$$-1A = -1 \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -1\cdot 1 & -1\cdot 2 \\ -1\cdot 3 & -1\cdot 4 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$$

Multiplication matrix:



2x2 order matrix:

$$\begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix} \times \begin{bmatrix} 6 & 5 \\ -7 & 1 \end{bmatrix} = \begin{bmatrix} -2\times6+1\times-7 & -2\times5+1\times1 \\ 0\times6+4\times-7 & 0\times5+4\times1 \end{bmatrix}$$

$$= \begin{bmatrix} -19 & -9 \\ -28 & 4 \end{bmatrix}$$

3x3 order matrix:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} (aj+bm+cp) & (ak+bn+cq) & (al+bo+cr) \\ (dj+em+fp) & (dk+en+fq) & (dl+eo+fr) \\ (gj+hm+ip) & (gk+hn+iq) & (gl+ho+ir) \end{bmatrix}$$

Properties of Matrix Multiplication:

There are certain properties of matrix multiplication operation in linear algebra in mathematics. These properties are as given below:

1) Non-Commutative: Matrix multiplication is non-commutative, i.e., for multiplication of two matrices A and B, AB ≠ BA.
2) Distributive: The distributive property can be applied while multiplying matrices, i.e., A(B + C) = AB + BC, given that A, B, and C are compatible.
3) Product with Scalar: If the product of matrices A and B, AB is defined then, c(AB) = (cA)B = A(Bc), such that c is a scalar.
4) Transpose: The transpose of the product of matrices A and B can be given as, (AB)T = BTAT, where T denotes the transpose.
5) Complex Conjugate: If A and B are complex entries, then (AB)* = B*A*
6) Associativity: Matrix multiplication is associative. Given three matrices A, B and C, such that the products (AB)C and A(BC) are defined, then (AB)C = A(BC).
7) Determinant: The determinant of product of matrices is nothing but the product of the determinants of individual matrices. i.e., det (AB) = det A × det B.

INVERSION OF MATRIX:

Inversion of matrix, Let A be a square matrix of order n. Then a matrix B, if it exists such that AB=BA=I is called inverse of the matrix A possesses an inverse then it must be unique.

- If determinant of | A |=0 is a singular matrix

- If determinant of | A |≠0 is non-singular matrix, there exists a inverse which is given by

$$A^{-1}=1/|A| (\text{adj } A)$$

Example:

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

SOLUTION OF LINEAR EQUATIONS USING MATRIX METHOD:

Solving linear equations using matrix is done by two prominent methods namely the matrix method and row reduction or Gaussian elimination method.

The first method to find the solution to the system of equations is a matrix method. The steps to be followed are:

- All the variables in the equation should be written in the appropriate order.
- The varibles, their coefficients and constants are to be written on the respective sides.

$$AX = B$$

Solving a system of linear equations by the method of finding the inverse consists of two new matrices namely,

- Matrix A: which represents the variables
- Matrix B: which represents the constants

A system of equations can be solved using matrix multiplication.

Example:

The system of equation is

$$2x + y + z = 1$$

$$x - 2y - z = \frac{3}{2}$$

$$3y - 5z = 9$$

Writing above equation as AX = B

$$\underset{A}{\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}}$$